

## Problem Set 3

ECON 340: Economic Research Methods

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Please show your work including the formula you used to calculate your answers.

1. (3 pts) Suppose you are given that  $X$  follows a normal distribution with mean 20 and variance 49 i.e.  $X \sim N(20, 49)$ .
  - (a) What is the probability that  $X$  is equal to 15?
  - (b) What is the probability that  $X$  is less than 15?
  - (c) What is the probability that  $X$  is between 15 and 25?
  - (d) The probability that  $X$  is more than  $x_0$  is 0.8. What is  $x_0$ ?
  - (e) Your friend is surprised that you were quickly able to solve complex integrals to answer these questions. Can you explain to your friend how you did it?
2. (3 pts) The table below gives us the joint probability distribution of time taken to read five pages of a book ( $Y$ ) and caffeine consumption ( $X$ ).

	Caffeine ( $X = 1$ )	No Caffeine ( $X = 0$ )	Total
10-mins ( $Y = 10$ )	0.2	0.1	0.3
20-mins ( $Y = 20$ )	0.3	0.4	0.7
Total	0.5	0.5	1

- (a) Find  $E(Y)$ , which is the (unconditional) expected value of time taken to read five pages.
- (b) Now calculate  $E(Y|X = 1)$  and  $E(Y|X = 0)$ . (Show your work.)
- (c) Given your answer in (b), are  $X$  and  $Y$  independent variables? Explain.
- (d) What can you say about the impact of caffeine on reading speed from your answer in (b)?

3. (2 pts) We have reasons to believe that the distribution of household income for the US population is right-skewed. I am thinking about taking a random sample of 10,000 individuals from the US population and calculating the mean.
- (a) Is it possible to ascertain whether the distribution of the sample mean in this case will be normal? If yes, how did you come to this conclusion?
  - (b) Say the true population mean of income is \$90,000 and the true standard deviation is \$68,000. What is the expectation and the variance of the sample mean in this case?
4. (2 pts) We took a random sample of 25 CSUF students and asked them how many hours did they spend in front of a screen in the past one week. Say we know that hours in front of the screen for CSUF students is normally distributed with unknown mean and variance  $\sigma^2 = 36$ . We found that average hours in front of a screen in our sample was 40. Construct a 95 percent confidence interval for the population mean.