Handout for Lecture 20

Calculus Review and Functional Forms

ECON 340: Economic Research Methods — Instructor: Div Bhagia

For a function $y = f(x)$, the derivative denoted by dy/dx or $f'(x)$ captures how the value of the function changes due to a small change in x.

Rules of differentiation:

(a) $y = a \rightarrow \frac{dy}{dx} = 0$

(b) $y = ax \rightarrow \frac{dy}{dx} = a$

(c) $y = ax^b \rightarrow \frac{dy}{dx} = abx^{b-1}$

(d) $y = f(x) \pm g(x) \rightarrow \frac{dy}{dx} = f'(x) \pm g'(x)$

(e) Derivative of a log function:

$$y = log(x) \rightarrow \frac{dy}{dx} = \frac{1}{x}$$

(f) Chain rule:

$$z = f(y),\ y = g(x) \rightarrow \frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Find the derivative for the following functions:

1. $y = 10 \rightarrow \frac{dy}{dx} = 0$
2. $y = 5x \rightarrow \frac{dy}{dx} = 5$
3. $y = 8x^3 \rightarrow \frac{dy}{dx} = 24x^2$
4. $y = 3x^2 + 4 \rightarrow \frac{dy}{dx} = 6x$
5. $y = 2 + 3 \cdot log(x) \rightarrow \frac{dy}{dx} = \frac{3}{x}$
6. $y = log(z),\ z = x^2 \rightarrow \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{z} \cdot 2x = \frac{2x}{x^2} = \frac{2}{x}$
7. $y = log(x^2) \rightarrow \frac{dy}{dx} = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$
8. $y = log(f(x)) \rightarrow \frac{dy}{dx} = \frac{f'(x)}{f(x)}$

- Find $\frac{dY}{dX}$ for the following model:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + u$$

What is the interpretation of β_1 and β_2?

$$\frac{dY}{dX} = \beta_1 + 2\beta_2 X$$

β_1 is the marginal effect of X on Y when X is equal to 0.

β_2 captures the change in the marginal effect of X on Y for each additional unit of X. In other words, for each one-unit increase in X, the effect of X on Y changes by $2\beta_2$.

- Consider the following model:

$$\log(Y) = \beta_0 + \beta_1 \log(X) + u$$

Differentiate both sides of the above equation with respect to X and show that β_1 represents the elasticity of Y with respect to X.

Differentiating the left-hand side, by chain rule we get:

$$\frac{d}{dX}\log(Y) = \frac{1}{Y} \cdot \frac{dY}{dX}$$

Differentiating the right-hand side :

$$\frac{d}{dX}(\beta_0 + \beta_1 \log(X) + u) = \beta_1 \cdot \frac{1}{X}$$

Setting the derivatives equal to each other since we are differentiating the same function:

$$\frac{1}{Y} \cdot \frac{dY}{dX} = \beta_1 \cdot \frac{1}{X}$$

Re-arranging the terms, we get:

$$\beta_1 = \frac{X}{Y} \cdot \frac{dY}{dX} \equiv \text{Elasticity of } Y \text{ with respect to } X$$