## Handout for Lecture 20

## Calculus Review and Functional Forms

ECON 340: Economic Research Methods

For a function $y=f(x)$, the derivative denoted by $d y / d x$ or $f^{\prime}(x)$ captures how the value of the function changes due to a small change in $x$.

Rules of differentiation:
(a) $y=a \rightarrow \frac{d y}{d x}=0$
(b) $y=a x \rightarrow \frac{d y}{d x}=a$
(c) $y=a x^{b} \rightarrow \frac{d y}{d x}=a b x^{b-1}$
(d) $y=f(x) \pm g(x) \rightarrow \frac{d y}{d x}=f^{\prime}(x) \pm g^{\prime}(x)$
(e) Derivative of a log function:
(f) Chain rule:

$$
y=\log (x) \rightarrow \frac{d y}{d x}=\frac{1}{x}
$$

$z=f(y), y=g(x) \rightarrow \frac{d z}{d x}=\frac{d z}{d y} \cdot \frac{d y}{d x}$

Find the derivative for the following functions:

1. $y=10 \rightarrow \frac{d y}{d x}=$
2. $y=5 x \rightarrow \frac{d y}{d x}=$
3. $y=8 x^{3} \rightarrow \frac{d y}{d x}=$
4. $y=3 x^{2}+4 \rightarrow \frac{d y}{d x}=$
5. $y=2+3 \cdot \log (x) \rightarrow \frac{d y}{d x}=$
6. $y=\log (z), z=x^{2} \rightarrow \frac{d y}{d x}=$
7. $y=\log \left(x^{2}\right) \rightarrow \frac{d y}{d x}=$
8. $y=\log (f(x)) \rightarrow \frac{d y}{d x}=$

Find $\frac{d Y}{d X}$ for the following model:

$$
Y=\beta_{0}+\beta_{1} X+\beta_{2} X^{2}+u
$$

What is the interpretation of $\beta_{1}$ and $\beta_{2}$ ?

Consider the following model:

$$
\log (Y)=\beta_{0}+\beta_{1} \log (X)+u
$$

Differentiate both sides of the above equation and show that $\beta_{1}$ represents the elasticity of $Y$ with respect to $X$.

