ECON 340
Economic Research Methods

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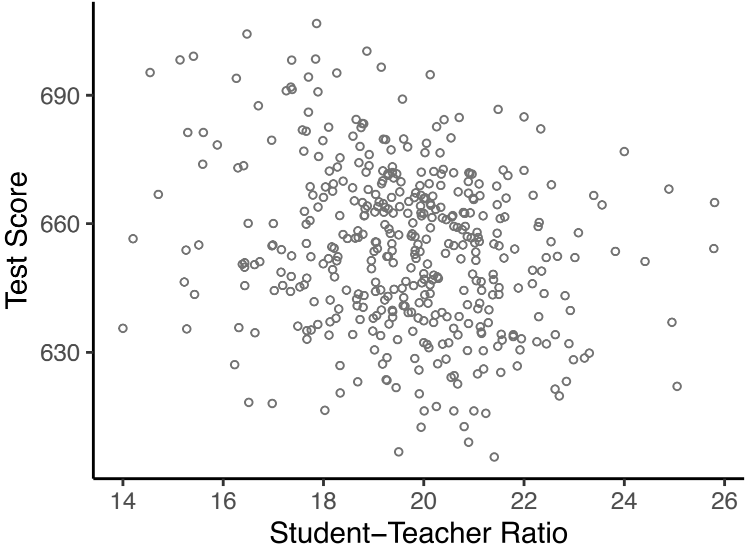
Lecture 15: Ordinary Least Squares, Goodness of Fit

Student-Teacher Ratio and Test Scores



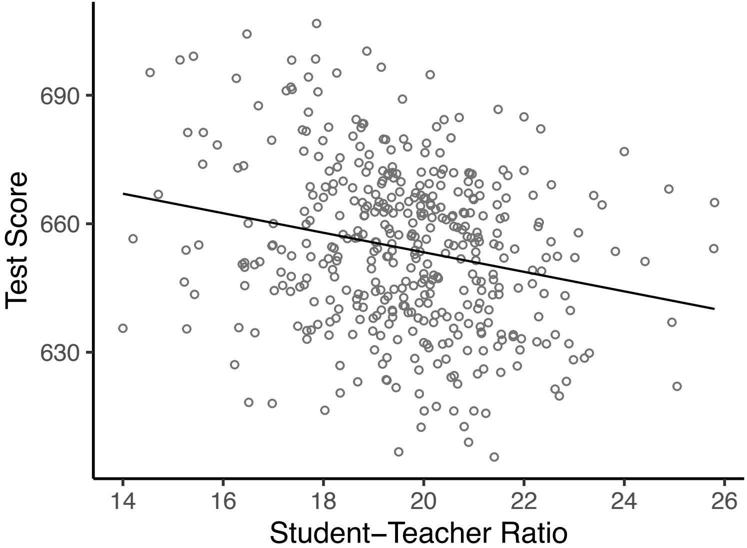
Fitting a Line

- We are interested in the relationship between two variables X and Y
- We start by assuming there is a linear relationship (with some error) between these variables in the population

$$Y = \beta_0 + \beta_1 X + u$$

- Fit a linear relationship between these two variables using sample data

Student-Teacher Ratio and Test Scores

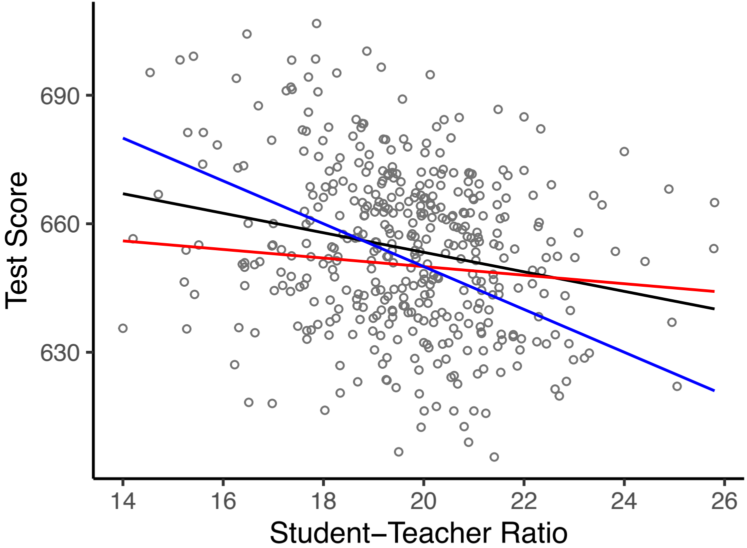


Simple Linear Regression Model

$$Y = \beta_0 + \beta_1 X + u$$

- Y: Dependent variable (outcome or response variable)
- X: Independent variable (explanatory variable, regressor)
- β_0, β_1: intercept and slope (population parameters)
- u: mean zero error term, $E(u) = 0$

Which is the best line?



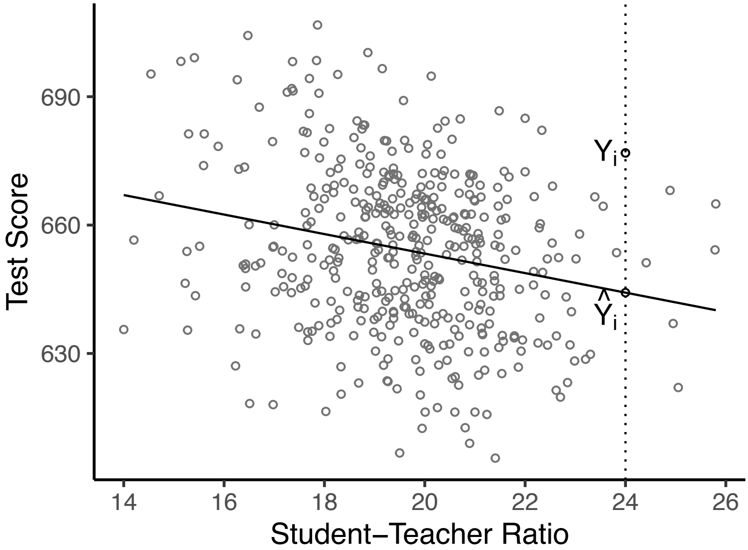
Ordinary Least Squares (OLS)

- We observe Y_i and X_i for all individuals in our sample.
- Find $\hat{\beta}_0$ and $\hat{\beta}_1$ from sample data by minimizing the sum of squared residuals

$$\min_{b_0, b_1} \sum_{i=1}^{n} (Y_i - b_0 - b_1 X_i)^2$$

- $\hat{\beta}_0$ and $\hat{\beta}_1$ are called ordinary least squares (OLS) estimators

Ordinary Least Squares (OLS)



Best line is the one that minimizes:

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

OLS Estimators

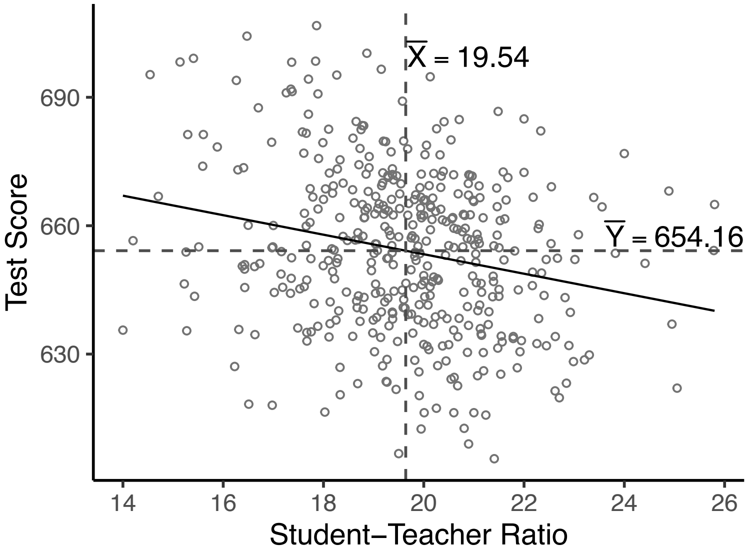
Some calculus reveals:

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{S_{XY}}{S_X^2}$$

$\rightarrow$ The best-fit line passes through the sample means!

OLS line passes through the means



Prediction and Residuals

OLS fitted line/predicted values:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

Residuals/prediction error (the one we minimized):

$$\hat{u}_i = Y_i - \hat{Y}_i$$

Prediction and Residuals

For our example, the fitted line:

$$\widehat{testscr} = 698.93 - 2.28 \cdot str$$

What is the predicted test score for a school with a student-teacher ratio of 24?

What is the prediction error for a school with a student-teacher ratio of 24 and average test score of 677?

How to interpret the coefficients?

Fitted line:

$$\widehat{testscr} = 698.93 - 2.28 \cdot str$$

- Intercept: Predicted test score is 698.93 for a school with $str = 0$. (Doesn't always make sense!)
- Slope: One more student per teacher lowers the predicted test score by 2.28. How?

  Alternatively: Schools in our sample that had one more student per teacher on average had an average test score that was 2.28 points lower.

Goodness of Fit: The R^2

- R-squared measures how well the OLS regression line fits the data
- R-squared is the percent of sample variation in Y that is explained by X

Note that:

$$Y_i = \hat{Y}_i + \hat{u}_i$$

R^2 is the ratio of sample variation of $\hat{Y}_i$ to sample variation of Y_i

Goodness of Fit: The R^2

Total Sum of Squares:

$$TSS = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

Explained Sum of Squares:

$$ESS = \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$$

Residual Sum of Squares:

$$RSS = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n}\hat{u}_i^2$$

Goodness of Fit: The R^2

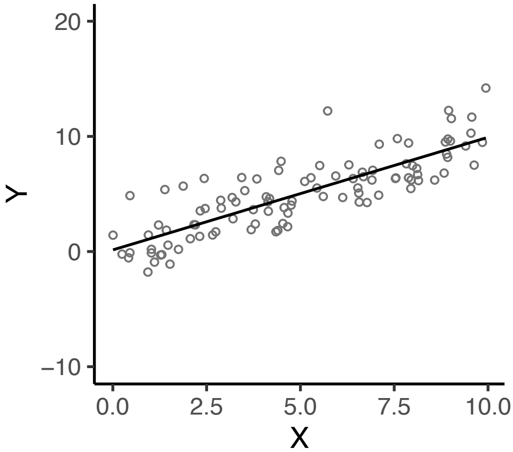
One can show that, $TSS = ESS + RSS$.

A measure of goodness of fit:

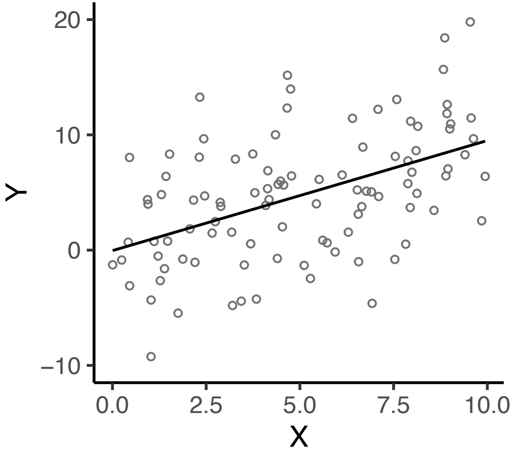
$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

Goodness of Fit: The R^2

High R^2



Low R^2



Goodness of Fit: The R^2

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

R^2 lies between 0 and 1

- If X explains no variation in Y, $\hat{\beta}_1 = 0$ and $\hat{Y}_i = \hat{\beta}_0 = \bar{Y}$. In which case, $ESS = 0$ and hence $R^2 = 0$.
- On the other hand, if X explains all the variation in Y, $\hat{Y}_i = Y_i$ and $RSS = 0$. In which case, $R^2 = 1$.