ECON 340
Economic Research Methods

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Lecture 14: Hypothesis Testing & p-Values

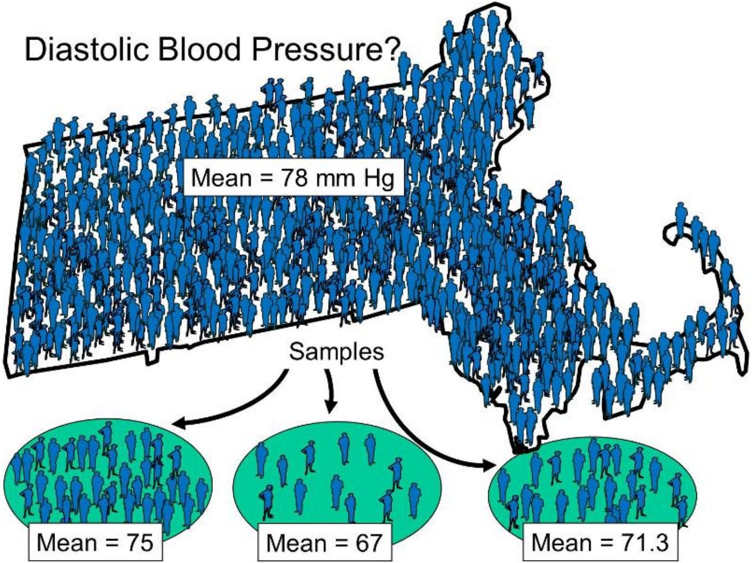
Sample Mean Distribution

Let $X_1, X_2, ..., X_n$ denote independent random draws (random sample) from a population with mean μ and variance σ^2.

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

The sample mean $\bar{X}$ is normally distributed when the underlying population is normal *or* sample size is large, say $n \geqslant 100$ (CLT).

Blood Pressure in Massachusetts



Blood Pressure in Massachusetts

Let's say we picked a random sample of 100 people from Massachusetts and took their blood pressure.

We found that the average diastolic blood pressure in our sample was 75 i.e. $\bar{x} = 75$

Let's say our prior was $\mu = 78$, but we found $\bar{x} = 75$. Can we conclude our prior was wrong?

Blood Pressure in Massachusetts

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Let's say our prior was $\mu = 78$, but we found $\bar{x} = 75$. Can we conclude our prior was wrong?

Not yet! It could be that the true mean was 78, but just by chance we got 75. We can formally test our "hypothesis."

Hypothesis Testing

- Note that $n = 100$. For now, assume we know $\sigma^2 = 552.25$.
- So if our initial hypothesis ($\mu = 78$) is correct, then

$$\bar{X} \sim N(78, 5.52)$$

- To test our hypothesis, say at 10% level of significance:
  - Find the 10% most surprising outcomes, assuming our initial hypothesis is true
  - If the obtained sample mean $\bar{x} = 75$ is in the 10% most surprising outcomes, agree that we were wrong

Hypothesis Testing

- If 75 is not in the 10% most surprising outcome.
- Then it is quite likely that with a true population mean of 78, we actually got a sample mean of 75.
- So we cannot conclude that our initial hypothesis was wrong at least under this criterion.
- To make this easy on ourselves, we will find the most surprising outcomes in terms of the standard normal distribution

Hypothesis Testing: Recipe

1. We will call our initial hypothesis as the **null hypothesis**.

$$H_0 : \mu = 78$$

We will test this against an **alternative hypothesis**. Natural alternative here is

$$H_1 : \mu \neq 78$$

Hypothesis Testing: Recipe

2. Given that our null is true, in this example we know that

$$\bar{X} \sim N(78, 5.52)$$

Test statistic under the null:

$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{x}}} \sim N(0, 1)$$

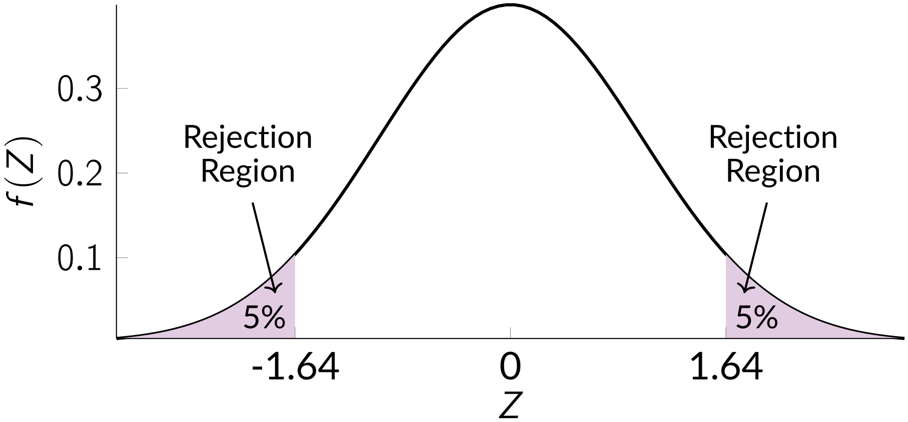
Hypothesis Testing: Recipe

3. **Significance level** α: determines how surprised do you have to be before you reject the null hypothesis.

   So our **rejection region** is $\alpha\%$ of outcomes that are most surprising given the null.

Rejection Region

At 10% level of significance:



Hypothesis Testing: Recipe

Here $z_{0.05} = 1.64$ is the **critical value**.

So if our test statistic

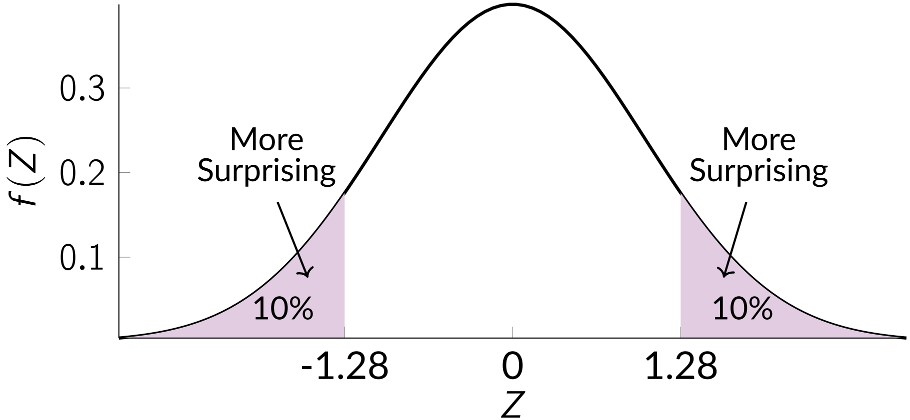
$$|z| = \left| \frac{\bar{x} - 78}{\sigma_{\bar{x}}} \right| > 1.64 \rightarrow \text{Reject the null}$$

Otherwise, do not reject the null.

In our example, we cannot reject the null at 10% level of significance as $|z| = \left| \frac{75-78}{\sqrt{5.52}} \right| = 1.28 < 1.64$.

p-Value

p-value is defined as the probability of randomly drawing an outcome as surprising or more surprising given the null.



p-Value: Recipe

$$\text{p-value} = 2P\left(Z > \left|\frac{\bar{x} - 78}{\sigma_{\bar{x}}}\right| \middle| H_0 : \mu = 78\right)$$
$$= 2P(Z > 1.28) = 0.2$$

If p-value $< \alpha$, we can reject the null at α level of significance.

What if we don't know σ^2

- Proceed as before and create a test statistic using S:

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$$

- Reject the null if $|t| > t_{n-1,\alpha/2}$
- For large n, say $n \geqslant 100$, standard normal distribution approximates the t-distribution well
- So with large samples, we calculate a t-statistic but can just look at the standard normal table for critical values

Next up

- Next class: Review for the midterm
- Midterm exam on Thurs. Study guide and sample exam uploaded on the course website.