Handout for Lecture 10

Normal Distribution and Z-Score

ECON 340: Economic Research Methods — Instructor: Div Bhagia

If $X \sim N(\mu, \sigma^2)$, then the standardized random variable,

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

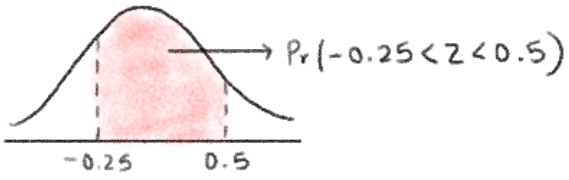
Given $X \sim N(\mu, \sigma^2)$, to find $Pr(x_0 < X < x_1)$:

- Find $z_0 = (x_0 - \mu)/\sigma$ and $z_1 = (x_1 - \mu)/\sigma$
- Use standard normal table to find $Pr(z_0 < Z < z_1)$

Exercises: Refer to the standard normal table to answer the following.

1. Given $X \sim N(3, 16)$, find $Pr(2 < X < 5)$.

$Pr(2<X<5) = Pr\left(\frac{2-3}{4} < Z < \frac{5-3}{4}\right) = Pr(-0.25<Z<0.5)$



From the standard Normal table:

$Pr(Z<-0.25) = 0.4013$

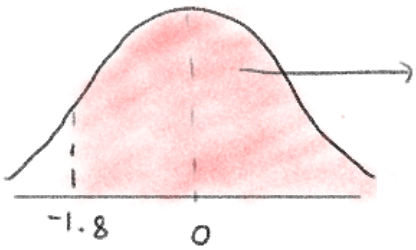
$Pr(Z<-0.5) = Pr(Z>0.5) = 0.3085$

So, $Pr(-0.25<Z<0.5) = 1 - 0.4013 - 0.3085$

$= \boxed{0.2902}$

2. Given $X \sim N(15, 100)$, find $Pr(X > -3)$.

$Pr(X>-3) = Pr\left(\frac{X-15}{10} > \frac{-3-15}{10}\right) = Pr(Z>-1.8)$



$Pr(Z>-1.8) = 1 - Pr(Z<-1.8)$

$= 1 - 0.0359$

$= \boxed{0.9641}$

Got this from the standard normal table

Alternatively, $Pr(Z>-1.8) = Pr(Z<1.8)$

$= 0.9641$

Given $X \sim N(\mu, \sigma^2)$ and $Pr(X < x) = p$, to find x :

- Use standard normal table to find z where $Pr(Z < z) = p$
- Find $x = \mu + z \cdot \sigma$

Follows analogously for when we are given $Pr(X > x) = p$.

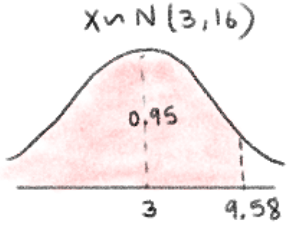
Exercises: Refer to the standard normal table to answer the following.

1. Given $Pr(Z > z) = 0.95$. Find z.

   From the standard normal table, $Pr(z < 1.645) = 0.95$
   Since the normal distribution is symmetric, $Pr(z > -1.645) = 0.95$
   So, $z = -1.645$

2. Given $X \sim N(3, 16)$ and $Pr(X < x) = 0.95$. Find x.

   From the standard normal table,
   $Pr(z < 1.645) = 0.95$
   Since, $z = \frac{x - \mu}{\sigma} \rightarrow x = \mu + z\sigma$
   $x = 3 + 1.645 \times 4 = 9.58$

3. Given $Pr(|Z| > z) = \sout{0.90}$ 0.10 (typo). Find z.

   Note: Since the normal distribution is symmetric $Pr(Z > z) = Pr(Z < -z)$, so we have that: $Pr(|Z| > z) = 2Pr(Z > z) = 2Pr(Z < -z)$.

   $Pr(|z| > z) = Pr(z < -z) + Pr(z > z) = 2\,Pr(z < -z)$
   Since $Pr(|z| > z) = 0.1$, trying to find z such that
   $Pr(z < -z) = 0.05$
   From standard normal table,
   $z = 1.645$