Summation Notation

ECON 340: Economic Research Methods   Instructor: Div Bhagia

The capital sigma (Σ) stands for summing everything on the right.

$$\sum_{i=1}^{N} X_i = X_1 + X_2 + \ldots + X_N$$

Things you CAN do to summations:

1. Pull constants out of them, or into them.

$$\sum_{i=1}^{N} b X_i = b \sum_{i=1}^{N} X_i$$

2. Split apart (or combine) sums (addition) or differences (subtraction)

$$\sum_{i=1}^{N} (b X_i + c Y_i) = b \sum_{i=1}^{N} X_i + c \sum_{i=1}^{N} Y_i$$

3. Multiply through constants by the number of terms in the summation

$$\sum_{i=1}^{N} (a + b X_i) = aN + b \sum_{i=1}^{N} X_i$$

Things you CANNOT do to summations:

1. Split apart (or combine) products (multiplication) or quotients (division).

$$\sum_{i=1}^{N} X_i Y_i \neq \sum_{i=1}^{N} X_i \times \sum_{i=1}^{N} Y_i$$

2. Move the exponent out of or into the summation.

$$\sum_{i=1}^{N} X_i^a \neq \left(\sum_{i=1}^{N} X_i\right)^a$$

<u>Exercise:</u>

$$X = \{2, 9, 6, 8, 11, 14\} \qquad Y = \{7, 1, 3, 5, 0\}$$

1. $\displaystyle\sum_{i=1}^{4} X_i = X_1 + X_2 + X_3 + X_4 = 2 + 9 + 6 + 8 = 25$

2. $\displaystyle\sum_{i=1}^{4} 2X_i = 2\sum_{i=1}^{4} X_i = 2 \times 25 = 50$

3. $\displaystyle\sum_{i=1}^{4} (X_i + 4) = \sum_{i=1}^{4} X_i + 4 \cdot 4 = 25 + 16 = 41$

4. $\displaystyle\sum_{i=1}^{3} (X_i + Y_i) = (X_1 + Y_1) + (X_2 + Y_2) + (X_3 + Y_3) = (2 + 7) + (9 + 1) + (6 + 3) = 28$

5. $\displaystyle\sum_{i=1}^{2} X_i Y_i = X_1 Y_1 + X_2 Y_2 = 2 \cdot 7 + 9 \cdot 1 = 23$

6. $\displaystyle\sum_{i=1}^{2} X_i \times \sum_{i=1}^{2} Y_i = (X_1 + X_2) \times (Y_1 + Y_2) = (2 + 9) \times (7 + 1) = 88$

7. $\displaystyle\sum_{i=1}^{2} X_i^2 = X_1^2 + X_2^2 = 4 + 81 = 85$