We can check for linear independence and find the rank of a matrix by converting the matrix to its echelon form.

How to convert a $3 \times 3$ matrix to its echelon form?

$$
A=\left[\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right]
$$

Step 1: Try to make $a_{31}=0$

$$
A_{1}=\left[\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
0 & a_{32} & a_{33}
\end{array}\right]
$$

Step 2: Try to make $a_{21}=0$

$$
A_{2}=\left[\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
0 & a_{22} & a_{23} \\
0 & a_{32} & a_{33}
\end{array}\right]
$$

Step 3: Try to make $a_{32}=0$

$$
A_{3}=\left[\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
0 & a_{22} & a_{23} \\
0 & 0 & a_{33}
\end{array}\right]
$$

Valid operations to convert to echelon form:

- Interchange any two rows
- Multiplication (or division) of a row by a scalar $k \neq 0$
- Addition of a (or $k$ times of a) row to another

Example.

$$
A=\left[\begin{array}{ccc}
1 & 3 & -4 \\
0 & 1 & 4 \\
-1 & 2 & 0
\end{array}\right]
$$

