Quasiconcave and Quasiconvex Functions

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Definitions

- A function is *quasiconcave* if and only if for any pair of distinct points u and v in the convex domain of f, and for $0 < \lambda < 1$, we have

$$f(\lambda u + (1 - \lambda)v) \geq \min\{f(u), f(v)\}$$

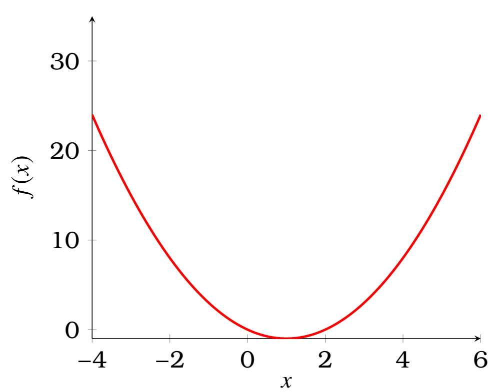
- A function is *quasiconvex* if and only if for any pair of distinct points u and v in the convex domain of f, and for $0 < \lambda < 1$, we have

$$f(\lambda u + (1 - \lambda)v) \leq \max\{f(u), f(v)\}$$

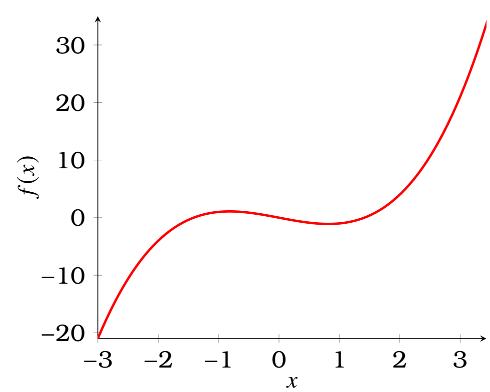
- Replace inequalities with strict inequalities to get the definitions of strict quasiconcavity and quasiconvexity.

Answer whether the following functions are (strictly) quasiconcave, (strictly) quasiconvex, both, or neither.

a)



b)



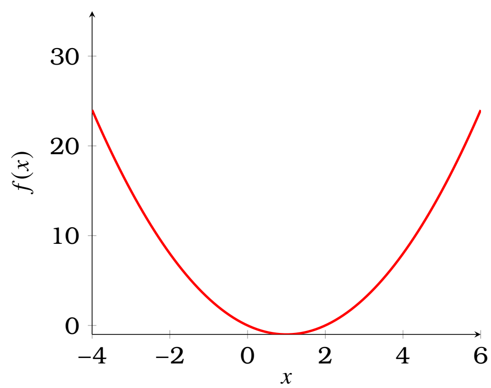
Alternative Definitions

- A function $f(x)$, where x is a vector of variables is *quasiconcave* iff for any constant k, the upper-contour set
$$S^U = \{x|f(x) \geq k\}$$
is a convex set.

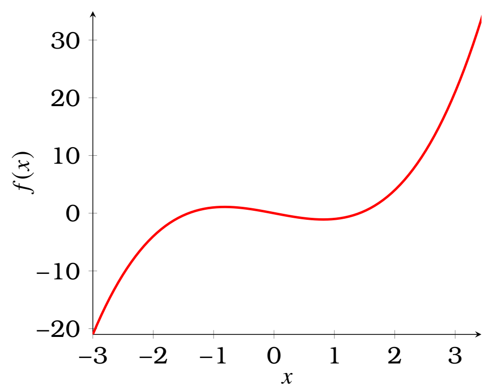
- A function $f(x)$, where x is a vector of variables is *quasiconvex* iff for any constant k, the lower-contour set
$$S^L = \{x|f(x) \leq k\}$$
is a convex set.

Use the alternative definitions to answer whether the following functions are (strictly) quasiconcave, (strictly) quasiconvex, both, or neither.

a)



b)



c) $f(x_1, x_2) = 6x_1 - 9x_2$