

## Midterm Spring 2024

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Print Name: \_\_\_\_\_

This is a closed-book test. You may not use a phone or a computer.

Time allotted: 110 minutes

Total points: 30

Please show sufficient work so that the instructor can follow your work.

*I understand and will uphold the ideals of academic honesty as stated in the honor code.*

Signature: \_\_\_\_\_

1. (6 pts) Answer the following questions.

(a) (1 pt) Consider a mapping  $f(x)$ . For two distinct values of  $x$ ,  $x_0$  and  $x_1$ ,  $f(x_0) = f(x_1)$ . Is  $f$  a valid function? Answer yes or no.

(b) (2 pts) Find the union and intersection for the following sets:

$$A = \{x : x \text{ is an even number}\} \quad B = \{2, 4, 8\}$$

(c) (1 pt) Consider the following two-variable function:

$$f(x, y) = x + y$$

where  $x \in (0, 1)$  and  $y \in (0, 1)$ . What is the range of  $f$ ?

(d) (1 pt) Given a system of linear equations  $Ax = b$ , if  $|A| = 5$ , what can we say about the solution for this system of equations?

- Has no solution.
- Has a unique solution.
- Has infinitely many solutions.
- None of the above

(e) (1 pt) Is the function  $y = |x|$  continuous at  $x = 0$ ? Answer yes or no.

2. (5 pts) Consider the following matrix

$$A = I - X(X'X)^{-1}X'$$

(a) (3 pts) Is  $A$  a square matrix? Show your work or reasoning that led you to this conclusion.

(b) (2 pts) Prove that  $A$  is idempotent i.e.  $AA = A$ .

3. (8 pts) Consider the following system of equations:

$$x - 2z = 2$$

$$y + z = 12$$

$$x + y + z = 24$$

(a) (1 pt) Write this system of equations in matrix format i.e.,

$$Av = b$$

What is  $A$ ,  $v$ , and  $b$  equal to?

(b) (2 pts) Calculate the adjoint of  $A$ .

(c) (2 pts) Calculate the determinant of  $A$ . Is  $A$  nonsingular?

(d) (1 pt) If you premultiply  $A^{-1}$  on both sides of the equation  $Av = b$ , you should be able to derive an expression to solve for  $v$ . Write down this expression.

(e) (2 pts) Using the expression in (d) solve for  $v^*$ .

4. (4 pts) Differentiate the following functions:

(a)  $y = 3x^3 + x^2 + 4$

(b)  $\frac{1}{x} + 3x^2$

(c)  $\frac{x-1}{x^2+3}$

5. (5 pts) Here is a demand function:

$$Q = 100 - 0.4p$$

where  $Q > 0$  is the quantity demanded and  $p > 0$  is the price.

(a) Calculate the elasticity of demand  $\varepsilon$  in terms of  $p$ .

(b) What is the elasticity at  $p = 50$ ? What about at  $p = 100$ ? Is demand elastic ( $|\varepsilon| > 1$ ) or inelastic ( $|\varepsilon| < 1$ ) at these prices?

- (c) Is the elasticity monotonically decreasing or increasing with price? (Note: I suggest taking the derivative of  $\varepsilon$  with respect to  $p$  instead of guessing.)

6. (2 pts) Say we have the following relationship between income ( $Y$ ), consumption ( $C$ ), and saving ( $S$ ).

$$Y = C + S$$

In addition, saving depends on interest rate  $i$  as follows:

$$S = g(i) + 100$$

Find the total derivative of income with respect to the interest rate.