

Spring 2023 Midterm Exam

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Print Name: _____

This is a closed-book test. You may not use a phone or a computer.

Time allotted: 110 minutes

Total points: 30

Please show sufficient work so that the instructor can follow your work.

I understand and will uphold the ideals of academic honesty as stated in the honor code.

Signature: _____

1. (8 pts) Answer the following questions (1 point each)

(a) Consider two sets A and B , where A is the set of all odd real numbers and B is the set of all real numbers. What is the intersection of A and B ?

(b) Expand the following summation expression: $\sum_{i=0}^3 (x + i)^2$

(c) Find the inverse of $f(x) = \frac{x-2}{3}$.

(d) Why do we need a matrix to be nonsingular when solving systems of linear equations?

- To ensure that the system of equations has a unique solution.
- To ensure that the system of equations has no solutions.
- To ensure that the system of equations has infinitely many solutions.
- It does not matter if the matrix is singular or nonsingular.

(e) Is the following function continuous? Is it differentiable?

$$f(x) = \begin{cases} 4 & \text{if } x < 2 \\ 10 & \text{if } x \geq 2 \end{cases}$$

(f) For the function $f(x) = \ln x$, $f'(x) = 1/x$

- True
- False

(g) Find the derivative of $y = \frac{1}{x}$.

(h) Find the derivative of $y = (2 - 3x)(1 + x)$.

2. (10 pts) Consider the following system of equations:

$$4x + 3y - 2z = 7$$

$$x + y = 5$$

$$3x + z = 4$$

(a) (1.5 pt) Write this system of equations in matrix format, i.e.,

$$Av = b$$

What is A , v , and b equal to?

(b) (3 pts) Calculate the adjoint of A .

(c) (2 pts) Calculate the determinant of A . Is A nonsingular?

(d) (1.5 pt) If you premultiply A^{-1} on both sides of the equation $Av = b$, you should be able to derive an expression to solve for v . Write down this expression.

(e) (2 pts) Using the expression in (d) solve for v^* .

3. (6 pts) Fun with Calculus!

(a) (3 pts) Demand for a good as a function of its price is given as follows:

$$Q(p) = p^{-\frac{1}{1+\alpha}}$$

Calculate the elasticity of demand with respect to price. (Note: You can also take the log of both sides of the equation and write $\ln Q = -\frac{1}{1+\alpha} \cdot \ln p$, and use that equation if you like.)

(b) (3 pts) Suppose that aggregate income Y and population P are given by:

$$Y(t) = \ln P(t), \quad P(t) = ae^{rt}$$

where c , a , and r are constants. t denotes time. Find the growth rate of income, which is given by the derivative of Y with respect to t .

4. (6 pts) Consider the following production function with two inputs, capital (K) and labor (L):

$$Q = 2K^{1/2}L^{1/2}$$

The marginal product of an input is given by the partial derivative of the production function with respect to that input variable.

- (a) (3 pts) Show that the marginal product of capital (MPK) and labor (MPL) for the above production function are given by:

$$MPK = \frac{1}{2} \cdot \frac{Q}{K} \quad MPL = \frac{1}{2} \cdot \frac{Q}{L}$$

- (b) (2 pts) Now, say that in equilibrium, wages (w) are equal to the marginal product of labor i.e.

$$w = \frac{1}{2} \cdot \frac{Q}{L} = K^{1/2}L^{-1/2}$$

Given $K = 100$, write labor demand L as a function of wages w . (Essentially, you are finding the inverse of a function).

- (c) (1 pt) Given your answer in (b), do you think labor demand increases or decreases with an increase in wages?